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13. ABSTRACT *(Maximum 200 words)*

New nonlinear signal processing and modeling techniques were examined. Three key issues formed the focus of the project: 1] the incorporation of symmetries into the modeling process. Such symmetires might be deduced from fundamental principles, or inferred from observations. The incorporation of such symmetries leads to simpler, more robust models, with fewer free parameters. 2] The design of coupling terms for synchronizing the model with driving signals from the system of interest. This is the first analytical result of its kind, and gives sufficient conditions for guaranteeing that the model will synchronize. 3] The successful use of symbolic time series analysis techniques to perform parameter estimation for spatio-temporal (distributed) systems. It was shown that the symbol statistics from a single site time series could be used for the parameter fitting even when the underlying system was turbulent..

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Signal, modeling, control, nonlinear, synchronization

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Improved Techniques for Modeling and Controlling Nonlinear Systems with Few Degrees of Freedom

Final Report for Grant No. AFOSR F49620-95-1-0261
from
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Executive Summary

Statement of objectives

The research carried out under this grant concerned the development of improved modeling and control techniques. The techniques are built upon observational data and are tailored to the problem at hand. The models used are global discrete time mappings, ordinary and partial differential equations.

One major effort of the proposed program is the exploitation of symmetries to constrain the fitted models. These models are more compact and can accurately model behaviors not explicitly shown in the data.

A second major effort is an examination of synchronization between identical dynamical systems. The focus has been on developing a method for designing coupling schemes that guarantee synchronous motion between the systems. These results are analytical and represent a method for determining the complete state of a nonlinear system from limited measurements.

A third major effort concerned the development of symbolic time series methods for parameter fitting in both low- and high-dimensional systems. This technique shows particular promise in high-noise situations, where it has been shown to be capable of robust parameter estimation.

Methods employed

The models used are expansions in polynomials, or have been derived from first principles. The basis set used for the expansion is constructed to be orthonormal on the measured data after embedding into an appropriate state space. The coefficients of the models are fit to the data by either a least squares, or an annealing procedure using the raw data or its coarse-grained symbolic form. In addition, the fitting procedure accounts for the size and complexity of the expansion models. This results in models that are optimal in the sense that they are the simplest models (within a given class of models) that are consistent with the data.

An important and novel aspect of the research program is the array of tests and constraints implemented to ensure that the models are correct and contain the right physics. In order to insure that the models are close approximations to the true equations of motion in the reconstructed phase space a series of *a priori* constraints and/or *a posteriori* tests are imposed. The *a priori* constraints involve determining from the data any symmetries the attractor may exhibit and restricting the class of models to those that respect the symmetries. The *a posteriori* test compares the coarse-grained symbol statistics generated by the model with that of the data.

Significance of the proposed activity

The proposed research will significantly enhance the ability to detect, model and control the dynamics of low-dimensional nonlinear systems using observed time series data. The synchronization of two nonlinear oscillators can be exploited as a means of nondestructive testing of devices, or as a real time monitor of dynamics, or as a mechanism for controlling dynamics. The approach used is comprehensive and will be implemented on experimentally obtained data from a diverse group of sources.

Contents

C. RESEARCH OBJECTIVES

The goal of this research program is the development of new robust algorithms for diagnosing, modeling, and controlling nonlinear systems that have only a few degrees of freedom. (In this context the number of *degrees of freedom* is equal to the number of independent variables needed to accurately model the system which generated the data.) While systems with a few degrees of freedom have been the focus of the research, an important result is that some of the techniques also are applicable to spatio-temporal (distributed) systems. These systems are typically modeled as partial differential equations and, in principle, have an infinite number of degrees of freedom.

Time-series data, in conjunction with whatever *a priori* information is known about the system, are used as input to the modeling process. These methods are robust to uncertainties in the models, and to the presence of noise [1–5]. These techniques have been successfully applied to data from chemical reactions, electronic circuits, and mechanical systems [1]. Other researchers are also beginning to apply these methods to experimental data [6–8].

D. BACKGROUND

Successful modeling relies on mathematical results from Mañé [9], Takens [10], Sauer, Yorke, and Casdagli [11], and others who have shown that it is often possible to reconstruct the full multidimensional dynamics of a nonlinear system from a *single* scalar time series, $s(n) = s(n\tau)$ for $n = 1, 2, \ldots$ (τ is the sampling interval associated with the measurements) [12]. The most common technique uses time delays to form vectors $\mathbf{y}(n)$

$$\mathbf{y}(n) = [s(n),\, s(n+T),\, \ldots, s(n+(d-1)T)],$$

where $\mathbf{y}(n) \in \mathbb{R}^d$. Time evolution, as given by $\mathbf{y}(n) \mapsto \mathbf{y}(n+1)$, is equivalent (diffeomorphic) to the true evolution of the system that produced the scalar time series. Fraser and Swinney [13] have developed a technique for determining T from data, while Kennel et. al. [14] have developed a technique for determining d.

1. Low-dimensional modeling

The dynamics is modeled by *global* discrete-time maps

$$\mathbf{y}(n+1) = \mathbf{F}[\mathbf{y}(n)],$$

or *global* ordinary differential equations (ODE's)

$$\frac{d\mathbf{y}}{dt} = \mathbf{F}(\mathbf{y}).$$

When modeling the observed system via ODE's, time evolution of the data is modeled as a single implicit Adams integration time step

$$\mathbf{y}(n+1) = \mathbf{y}(n) + \tau \sum_{j=0}^{M} a_j^{(M)} \mathbf{F}[\mathbf{y}(n+1-j)].$$

The $a_j^{(M)}$'s are the Adams integration coefficients, and are known for all values of j and M.

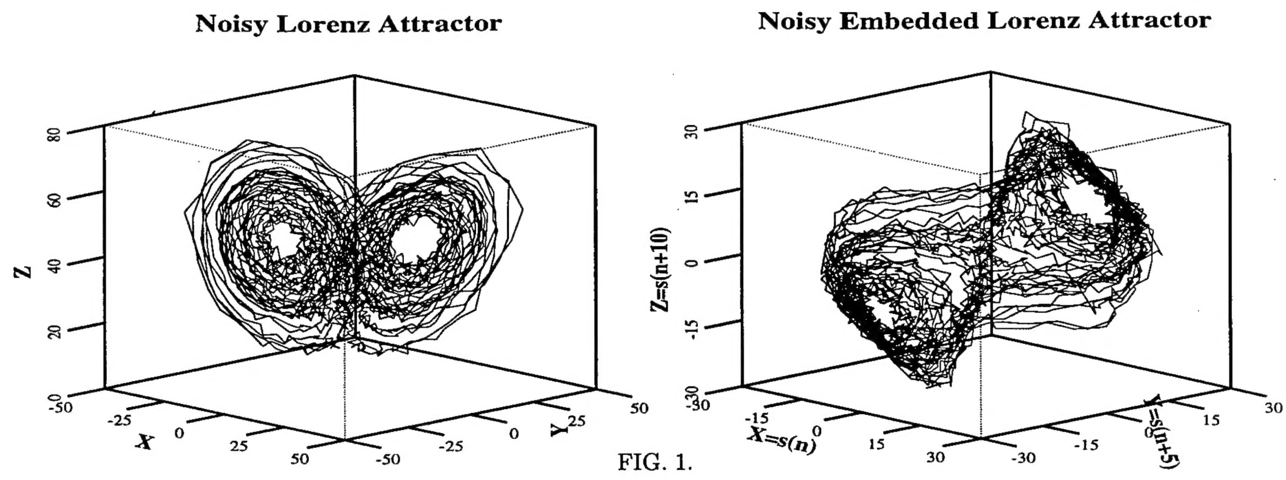

FIG. 1.

For a generic time series one does not know the functional form of $\mathbf{F}$. Hence, the best that one can hope for is a series expansion in some set of basis functions, $\phi^{(\mathbf{I})}(\mathbf{z})$,

$$\mathbf{F}(\mathbf{z}) = \sum_{\mathbf{I}=0}^{\mathbf{Np}} \mathbf{p}^{(\mathbf{I})} \phi^{(\mathbf{I})}(\mathbf{z}).$$

The data approximates an invariant distribution on the attractor that defines the system under observation

$$\rho(\mathbf{z}) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \delta[\mathbf{z} - \mathbf{y}(n)],$$

where N is the number of data vectors. The basis functions $\phi^{(\mathbf{I})}(\mathbf{z})$, are polynomials constructed (via Gram-Schmidt) to be orthonormal on the attractor defined by the data,

$$\begin{aligned} \left\langle \phi^{(\mathbf{I})} | \phi^{(\mathbf{J})} \right\rangle &= \int d\mathbf{z}\, \rho(\mathbf{z})\, \phi^{(\mathbf{I})}(\mathbf{z})\, \phi^{(\mathbf{J})}(\mathbf{z}) \\ &= \delta_{\mathbf{I}\,\mathbf{J}}. \end{aligned}$$

In this equation $\mathbf{I}$ and $\mathbf{J}$ are index vectors used to indicate the order of the polynomials.

It has been shown that this formulation of the modeling problem allows more accurate and robust determination of $\mathbf{F}$, for larger sampling intervals, than previous methods [1,7,8,15–17].

E. NEW RESULTS

1. Using symmetries to constrain expansion models

Symmetries represent an important class of physical properties that can be used to constrain models of dynamical systems. Using a symmetry to constrain a model means restricting the functional form of the model to one that, *a priori*, contains the symmetry. They can be determined from the data and/or from first principles. We have found that if a symmetry is present in the data or the dynamics then an equivariante model constructed from the data accurately models the dynamics [2].

To illustrate how symmetries constrain a model, suppose the attractor, A, is invariant under the action of the symmetry $\mathbf{S}$ ($\mathbf{S} \circ A = A$). This condition results in the following equivariance condition on $\mathbf{F}$

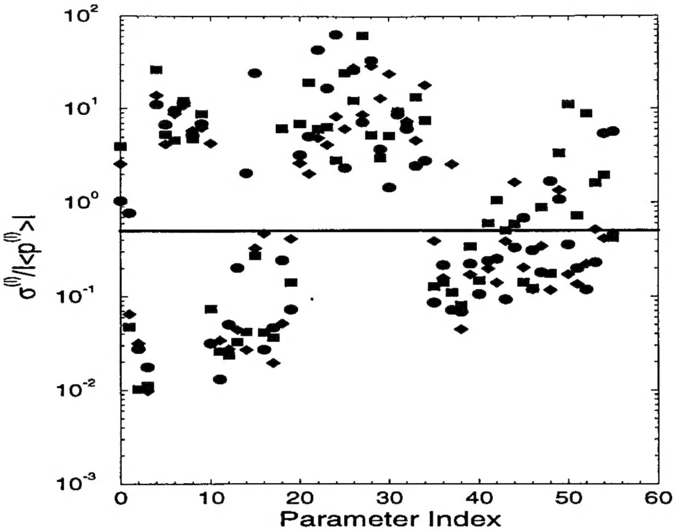

FIG. 2. Graphs of parameter uncertainty versus the index associated with the parameter.

$$\mathbf{F}(\mathbf{S} \circ \mathbf{y}) = \mathbf{S} \circ \mathbf{F}(\mathbf{y}). \tag{1}$$

Clearly, Eq. (1) places restrictions on the functional form of $\mathbf{F}$.

The Lorenz system is a dynamical system where we used symmetry to restrict the functional form of $\mathbf{F}$. Figure 1(a) shows a noisy representations of the true Lorenz attractor. Despite the noise, Fig. 1(a) indicates that if $\mathbf{S}$ is a rotation about the $\hat{z}$ axis by π radians then $\mathbf{S} \circ A = A$. The origin of this symmetry is the $x \to -x$ and $y \to -y$ invariance of the Lorenz equations. For this system Eq. (1) implies that all terms proportional to $x^n y^m$ must vanish for even values of $n + m$. Thus, on the basis of equivariance half of the terms in the model can be eliminated. Of course, if the data $\mathbf{y} = [x, y, z]$ is noise free then we expect the coefficients of these terms to vanish. However, *experimental data is never noise free.* Therefore, it is best to, *a priori*, eliminate terms from the model.

Figure 1(b) shows a Lorenz attractor that has been reconstructed by embedding a scalar time series given by the x coordinate of the data shown in Fig. 1(a). The time series is symmetric about zero which leads to the inversion symmetry ($\mathbf{S} = -\mathbf{1}$) evident in Fig. 1(b). Using an ensemble of data sets we constructed models without the restriction of Eq. (1). The results are shown in Fig. 2, where $\left\langle \mathbf{p}^{(\mathrm{I})} \right\rangle$ is the mean value of the parameter $\mathbf{p}^{(\mathrm{I})}$ and $\boldsymbol{\sigma}^{(\mathrm{I})}$ is the standard deviation of the parameter about its mean value. The circles squares and diamonds indicate the first, second, and third components of $\mathbf{p}^{(\mathrm{I})}$, respectively. The solid line represents $\boldsymbol{\sigma}^{(\mathrm{I})}/|\left\langle \mathbf{p}^{(\mathrm{I})} \right\rangle| = 2$.

For symbols above this line $|\left\langle \mathbf{p}^{(\mathrm{I})} \right\rangle| \pm \boldsymbol{\sigma}^{(\mathrm{I})}$ straddles zero. Hence, the value of $\mathbf{p}^{(\mathrm{I})}$ obtained by the fitting procedure is statistically indistinguishable from zero and one conjectures that their values are dominated by noise, finite sample size, and round off effects. This is supported by noticing that *all* of the symbols for index values 0, 4–9, and 20–34 are above the line. These indices correspond to terms in $\mathbf{F}$ whose coefficients, $\mathbf{p}^{(\mathrm{I})}$, should vanish because their basis function, $\phi^{(\mathrm{I})}$, is not equivariant under $\mathbf{S} = -\mathbf{1}$.

For comparison we also estimated the uncertainty of the $\mathbf{p}^{(\mathrm{I})}$'s using standard statistical methods [2]. These uncertainties, $\boldsymbol{\mu}^{(\mathrm{I})}$, were found to be good estimates of $\boldsymbol{\sigma}^{(\mathrm{I})}$. Thus, in some cases, it is possible to detect the presence of a symmetry from a single data set by examining $\boldsymbol{\mu}^{(\mathrm{I})}/|\left\langle \mathbf{p}^{(\mathrm{I})} \right\rangle|$ [2].

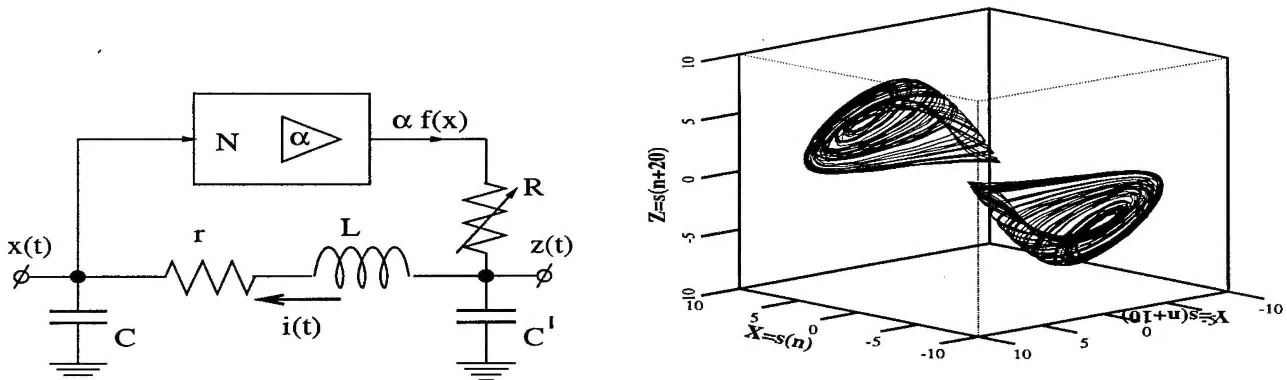

FIG. 3.

We also examined data from the electronic circuit shown in Fig. 3(a). It is possible to determine from first principles that, for certain values of α, the dynamics lives on one of the two disjoint attractors shown in Fig. 3(b) [18]. For this value of α a clear plane of symmetry can be seen between the attractors and a single experimentally measured time series would correspond to one or the other of these attractors, but not both. As α increases the two attractors merge into a single attractor via a symmetry increasing bifurcation (also called a crisis [19]).

Our work examined data from before and after the bifurcation. When examining data taken before the bifurcation we found that equivariant models constructed using data from one of the attractors accurately mimics the dynamics on *both* attractors [2]. This is not the case for previous modeling techniques, which require data from both attractors and two models [2]. Furthermore, the equivariant model contains fewer coefficients than either of the nonequivariant models. When modeling data taken after the bifurcation we found that we were able to detect the presence of the symmetry using the methods discussed above for the Lorenz example.

a. Synchronization and the observer problem

Synchronization between a model and a physical system is a solution to the observer problem for nonlinear plants. This approach couples the dynamics of the nonlinear plant to a model via drive–response coupling

$$\begin{aligned}\frac{d\mathbf{x}}{dt} &= \mathbf{F}(\mathbf{x};t)\\ \frac{d\mathbf{y}}{dt} &= \mathbf{F}(\mathbf{y};t) - \mathbf{E}\cdot(\mathbf{y}-\mathbf{x}),\end{aligned} \tag{2}$$

where $\mathbf{x} \in \mathbb{R}^d$ represents the dynamics of the plant and $\mathbf{y} \in \mathbb{R}^d$ represents the dynamics of the model. (Here we ignore modeling errors, having examined these issues in a previous paper [20].) The coupling, $\mathbf{E}$, is a vector function of its argument and $\mathbf{E}(\mathbf{0}) = \mathbf{0}$. Evaluating $\mathbf{E}$ does not necessarily require all components of $\mathbf{x}$. Because the models are deterministic we know that if $\mathbf{y} = \mathbf{x}$ at any one time then $\mathbf{y} = \mathbf{x}$ for all subsequent times. (The dynamics of Eqs. (2) is said to be synchronized if $\mathbf{x} = \mathbf{y}$.) Thus, if one can determine $\mathbf{E}$ such that $\lim_{t\to\infty} |\mathbf{y} - \mathbf{x}| = 0$ then one can determine, $\mathbf{x}$, the complete state of the plant by obtaining $\mathbf{y}$ from the model.

The major result of our research is a rigorous criteria which, if satisfied, guarantees that the invariant manifold given by $\mathbf{x} = \mathbf{y}$ is linearly stable. More importantly, the criteria can be used to

design couplings, $\mathbf{E}$, which satisfy the criteria. The criteria only uses knowledge of the uncoupled dynamics, $\mathbf{F}$, and many of the important calculations can be performed analytically [3].

Given the following optimal decomposition [3]

$$\mathbf{A} = \langle \mathbf{DF} \rangle - \mathbf{DE}(\mathbf{0}) \tag{3}$$

the criteria for linear stability of the manifold $\mathbf{x} = \mathbf{y}$ is [3]

$$-\Re[\Lambda_1] > \left\langle \|\mathbf{P}^{-1} [\mathbf{DF}(\mathbf{x}) - \langle \mathbf{DF} \rangle] \mathbf{P}\| \right\rangle, \tag{4}$$

where Λ_1 is the eigenvalue of $\mathbf{A}$ that has the largest real part, $\Re[\Lambda]$ is the real part of Λ, and $\mathbf{P}$ is a matrix composed of the eigenvectors of $\mathbf{A}$. In Eqs. (3) and (4), $\langle \bullet \rangle$ denotes a time average along the driving trajectory, $\mathbf{x}$.

Equations (3) and (4) represent definitions and conditions that indicate when synchronous motion along a particular driving trajectory becomes unstable to small perturbations in directions transverse to the synchronization manifold. The criterion is rigorous and sufficient. However, because it is based on norms it is not necessary.

Because the integral in Eq. (4) is positive semi-definite the inequality can not be satisfied unless $\Re[\Lambda_1] < 0$. This condition is reminiscent of the discussion of stability of linear systems. In addition, it can be shown that up to second order the criteria for linear stability is $\Re[\Lambda_1] < 0$ [3].

The stability criteria depends explicitly on the measure of the driving trajectory. Work by many authors indicates that the most unstable trajectories are likely to be those associated with fixed points of $\mathbf{F}$ and unstable periodic orbits of $\mathbf{F}$ with the shortest periods [21–25]. Given these observations we conjecture that if these trajectories are stable then the manifold $\mathbf{x} = \mathbf{y}$ should be stable for all $\mathbf{x}$ [3].

Equation (4) has a geometrical interpretation that can be used to design couplings that yield stable synchronous motion. Both sides of Eq. (4) are functions of the elements of $\mathbf{DE}(\mathbf{0})$. Thus, $\left\langle \|\mathbf{P}^{-1} [\mathbf{DF}(\mathbf{x}) - \langle \mathbf{DF} \rangle] \mathbf{P}\| \right\rangle = \text{const.} \equiv C_1$ defines, $\Sigma_{\mathbf{x}}$, a family of surfaces in the parameter space of the elements of $\mathbf{DE}(\mathbf{0})$. In the same parameter space $-\Re[\Lambda_1] = \text{const.} \equiv C_2$ defines, Σ_Λ, a different family of surfaces. Therefore, the boundary of that portion of parameter space that yields linearly stable synchronous motion is the intersection of the family of surfaces $\Sigma_{\mathbf{x}}$ with the family of surfaces Σ_Λ. This approach is analytically shown in our manuscripts [3].

2. Symbolic time series analysis

In symbolic time-series analysis the state space of the system is partitioned into a finite number of cells and a symbol, s, is assigned to each cell. Such a symbolic approach is appealing, since the symbol statistics is robust in the presence of noise. Our previous work [4] showed that the symbolic data are quite robust and that even at high noise levels (signal/noise$\approx \mathcal{O}(1)$) the effects of noise on the symbolic data can be tracked and effectively eliminated.

To briefly summarize the approach, we convert the analog signal stream into a symbol sequence by passing it through a threshold function which takes $\{x_n\} = (x_1, x_2, \cdots, x_N) \rightarrow \{s_n\} = (s_1, s_2, \cdots, s_N)$ where $s_k \in (0, 1)$. For example, if $x_n < x*$ then $s_n \equiv 0$ and if $x_n > x*$ then $s_n \equiv 1$. From the symbol sequence, $\{s_n\}$, we construct the *symbol tree*

$$\begin{array}{cccccccc} & & p_0 & & & & p_1 & \\ & p_{00} & & p_{01} & & p_{10} & & p_{11} \\ p_{000} & p_{001} & p_{010} & p_{011} & p_{100} & p_{101} & p_{110} & p_{111} \end{array}$$

etc.

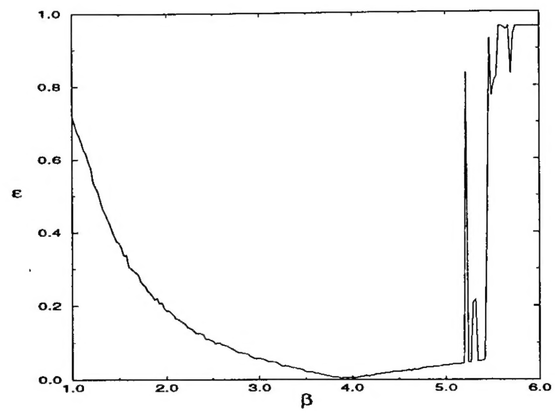

FIG. 4.

Here p_{001} is the probability of observing the sequence 001, etc. The symbol tree is a compact summary of (coarse grained) information about multiple-time-step correlations in the signal. Our goal is to construct a phase space evolution rule $(\mathbf{F}(\mathbf{y}; \vec{\lambda}))$ which generates the same symbol tree down to some level ($\vec{\lambda}$ represents the parameters in the model which can be varied).

The parameter fitting is done by varying the model dynamics such that, $\mathcal{E}(\vec{\lambda})$, the "distance" between the observed symbol tree and the tree generated by the model is minimized (see [4,5] for details). A plot of $\mathcal{E}(\vec{\lambda})$ vs $\vec{\lambda}$ constitutes the *error landscape* and reconstruction amounts to finding the global minimum in this landscape.

This approach has successfully been applied to fit parameters in spatio-temporal systems. Specifically [5] turbulent solutions of the complex Ginzburg-Landau equation

$$\frac{\partial A}{\partial t} = A + (1 - i\alpha)\nabla^2 A - (1 - i\beta)|A|^2 A$$

in both 1 and 2 spatial dimensions were generated with target values chosen for the parameters $\vec{\lambda}_0 = (\alpha_0, \beta_0)$. Time-series were then extracted from a *single* spatial site and used to construct the symbol tree. Fitting was carried out using only the symbol statistics. The error landscape for the fitting is shown in Fig. 4. As can be seen, there is a global minimum at the target value of $\beta_0 = 4.0$. An annealing calculation was then carried out and proved capable of estimating the parameters which generated the turbulent signal to within $2 - 3$ significant figures [5].

F. ACCOMPLISHMENTS AND NEW FINDINGS

The primary accomplishments of the grant period (4/1/95 - 9/30/96) are:

1. Low-dimensional modeling

   - Exploitation of symmetries to constrain the modeling process. Modeling algorithms were developed which detect and impose symmetries on the functional form of the model. This has been shown to improve the stability of the modeling process, as well as ensuring that important properties of the dynamical system are represented by the model *even if they are not directly present in the data set* [2].
   - A rigorous sufficient criteria has been derived which guarantees linearly stable synchronization between dynamical systems when they are coupled in a drive–response manner. This result is an analytic method for solving the observer problem for nonlinear plants [3].

2. Symbolic time series analysis

   - Demonstration that the symbol sequence statistics can be used as a target for reconstruction for spatio-temporal systems. Exhaustive numerical evidence was developed which indicates that the approach is both robust and convergent [5].

G. PERSONNEL SUPPORTED

Senior Personnel
E. R. Tracy (1 month).
Reggie Brown (3 months).

Post-doctoral fellow
Xian-zhu Tang (began 9/1/95).

H. PUBLICATIONS

1. R. Brown and N. F. Rulkov, "Synchronization of chaotic systems: transverse stability of trajectories in invariant manifolds", submitted for publication.
2. R. Brown, V. In and E. R. Tracy, "Parameter uncertainties in models of equivariant dynamical systems", to appear in Physica D.
3. X.-Z. Tang, E. R. Tracy and R. Brown, "Symbol statistics and spatio-temporal systems", to appear in Physica D.
4. R. Brown, "Using Models to diagnose, test, and control chaotic systems", to appear in the proceedings of the Third Experimental Chaos Conference.
5. G. Gouesbet, L. Le Sceller, C. Letellier and R. Brown, "Reconstruction of a set of equations from scalar time series", to appear in the proceedings of the Eleventh Annual Florida Workshop on Nonlinear Astronomy.

I. INTERACTIONS/TRANSITIONS

1. Presentations at meetings

1. Poster presentation (contributed) at Sherwood (Fusion Theory) Meeting, Lake Tahoe NV (April, 1995), E. Tracy.
2. Contributed Talk, Third SIAM Conference on Applications of Dynamical Systems", (May 1995) Reggie Brown.
3. Invited Talk, Third Experimental Chaos Conference, Edinburgh, SCOTLAND (August 1995) Reggie Brown.
4. Poster presentation (contributed) at American Physical Society, Division of Plasma Physics Meeting, Lousiville, KY (October, 1995), E. Tracy.
5. Seminar, U. of Richmond, Physics Department (November, 1995), E. Tracy.
6. Contributed Talk, Dynamics Days Conference, Houston, TX (January, 1996) X. Tang.
7. Colloquium, College of Wm. & Mary, Physics Department, (February, 1996) Reggie Brown.
8. Seminars were also given at AFOSR at DOE (December, 1995).

1. Wm & Mary/AlliedSignal:
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Contact:
Dr. R. Burne
Research Manager
(410)964-4159.
Anticipated result: Using the symbol statistics to detect transitions in complex systems, *e.g.* noise-driven turbulent flows.
Application: Early detection of rotating stall in turbines.

2. Oak Ridge National Lab/Ford Motor Company:
Work performed as part of a Cooperative Research and Development Agreement (CRADA), number ORNL-95-0337 titled "Engine Control Improvement Through Application of Chaotic Time Series Analysis".
Performer:
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Dr. John Hoard
(313)594-1316.

Anticipated result: Using symbol statistics to do parameter fitting for an internal combustion engine model. (Aspects of the work are subject to a patent disclosure.)
Application: Improved feedback control for internal combustion engines to reduce NOx emission and increase fuel efficiency.

J. BIBLIOGRAPHY

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